CONNECTED

lesson guide

Driving Question	Total Time
Do social networks make us more connected?	110 minutes

Real-World Takeaways

- As a social network grows, the number of possible connections among users grows by more and more
- With very large networks like Facebook, there are quintillions of possible connections among users
- Since Facebook users interact mostly with their friends, not all of these possible connections are realized
- While Facebook allows us to connect with new ideas, it also allows us to disconnect with ideas we don't agree with

Math Objectives

- Given a pattern, create a rule to determine the next value in the sequence
- Write and graph a quadratic function to model a real-world scenario; interpret the meaning of a non-linear rate of change
- Evaluate the quadratic function for different values of x
- Given a calculator result in scientific notation, understand this in "common language" (e.g. 1.4E9 = 1.4 billion)

Question Overview & Suggested Time

Preview		Watch first-ever commercial for Facebook and discuss reasons people use social networks	5 minutes
Act One	1	For a given number of users, determine how many connections are possible in network	25 minutes
	2	Write expression to determine total possible connections in 10-, 20-, and 100-person network	25 minutes
	3	Write and graph quadratic equation to model relationship between possible connections and number of users	20 minutes
Act Two	4	Use Facebook's user numbers to determine how total number of possible connections has changed over time; discuss difference between possible connections and actual connections	20 minutes
	6	Discuss whether Facebook results in users becoming more or less open-minded and consider impact on society	15 minutes

P Students watch the first-ever commercial for Facebook, which compares itself to chairs: places where people can sit down and *connect* with one another. Students then discuss their experiences with social networks. In particular, they discuss why social networks (Facebook, Snapchat, Instagram, LinkedIn, etc.) exist, and why so many people use them.

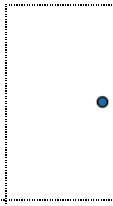
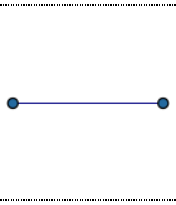
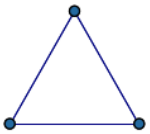
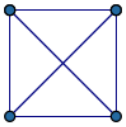
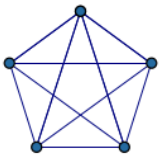
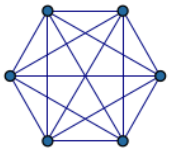
Discussion Questions

1. *In addition to Facebook, what are some other examples of social networks? Which (if any) do you use?*
2. *Why do you think people use social networks? What makes one social network better than another?*
3. *What do you think is the ultimate goal or purpose of a social network?*

Key Takeaway(s)

- Social networks connect users to one another. People use social networks to connect with their friends, family, etc.
- At the same time, social networks are also businesses that exist to make money.

1 Networks like telephone systems, Skype, Facebook, etc., depend on having lots of users; the more users there are, the more vibrant the network can be. The diagram below shows a network as it grows from one user to six. For each step, determine the **total number of possible connections** that can be made between users.

						
Users	1	2	3	4	5	6
Connections	0	1	3	6	10	15

Key Insights & Guiding Questions

- In the diagram, each point represents a user. A connection between two users can be represented by a segment, which can be thought of as a "conversation" or a "handshake" between two users.
  In the diagram, what does a point represent?
  If you draw a segment between two users, how could you describe what it means?

- As the number of users increases, the number of connections goes up by more and more.
  If you graphed this relationship between users and connections, how would you expect it to look?

- When a new user joins, they form a connection with every previous/existing user, so the number of previous users is equal to the number of *additional* connections brought by an additional users.
  How is the number of connections changing each time? What determines the number of "new" connections?

2 Without drawing anything, explain how you could determine the total number of connections in a network with ten users. Based on this, how many connections are possible in a network with 20 users? With 100 users?

Method 1: Summation	Method 2: Unique Connections per User
With 5 users, there were 4 + 3 + 2 + 1 = 10 connections. With 6 users, there are 5 + 4 + 3 + 2 + 1 = 15 connections. When a new user joins, he connects to everyone already in the network. If she's the 5th user, she'll add 4 new connections. If she's the 6th user, she'll add 5 new connections. Therefore, the number of connections for a given number of users is the sum of all of the previous numbers of users. 10 users: 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 45 connections 20 users: 19 + 18 + 17 + ... + 1 = 190 connections 100 users: 99 + 98 + 97 + ... + 1 = 4,950 connections	In a 6-person network, each person is connected to 5 other people. This makes for 6 • 5 = total 30 connections. However, Person A's connection to Person B (A-B) is the same as Person B's connection to Person A (B-A). Since every connection is counted twice, I must divide the 30 connections in half to get the number of unique connections. 10 users: (10 • 9) ÷ 2 = 90 ÷ 2 = 45 connections 20 users: (20 • 19) ÷ 2 = 380 ÷ 2 = 190 connections 100 users: (100 • 99) ÷ 2 = 9900 ÷ 2 = 4,950 connections

Teaching Tip

The first part of the question doesn't ask how many connections correspond to 10 users but rather *how students would determine* the number of connections; the emphasis isn't on the answer but on the strategy. To help students better understand the relationship between the number of users and the total connections, consider asking for volunteers to act out the situation. With one person standing, ask how many handshakes are possible (zero). When a second person joins the network, one handshake is possible: A-B. When a third person joins, three handshakes are possible: A-B, A-C, and B-C.

There are two goals of the "handshake game." The first is for students to recognize that when A shakes hands with B, B also shakes hands with A; it's the same handshake and should only be counted once. The second goal is for students to recognize a pattern in how the number of connections grows *and to be able to explain why*.

Key Insights & Guiding Questions

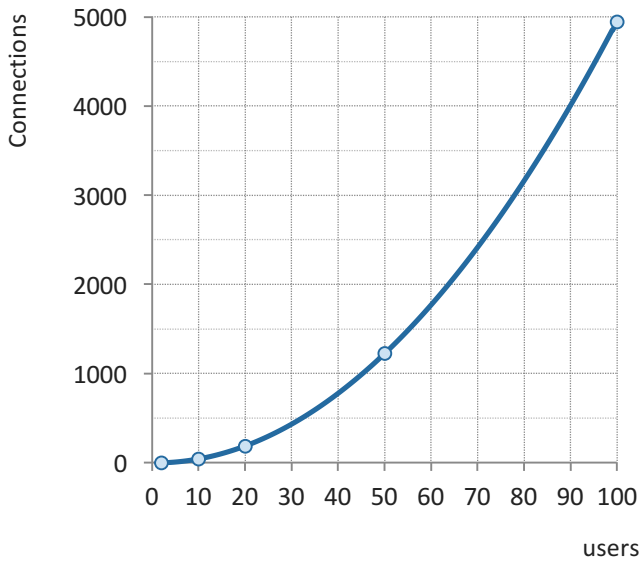
- As the number of users increases, the number of connections does not change by a constant amount; rather, it increases by more and more. This means the relationship is not linear.
  As the number of users increases, does the number of connections change by the same amount?
  If you were to graph the relationship between connections and users, what do you think it would look like?

- When a new person joins the network, she connects with each of the existing users. If she's the n^{th} user, she'll connect with (n – 1) existing users and will therefore contribute (n – 1) new connections.
  If there are nine users in the network, and a 10^{th} person joins, how many new connections will be made?

- The total number of connections corresponding to n users can be found by summing 1 through (n – 1). While the summation method works for small numbers of users like 10, it is cumbersome to use with large numbers of users.
  If the 10^{th} user contributes 9 new connections, how would you find the total connections in a 10-person network?
  Would you want to use the summation method to determine the number of connections for 100 users?

- In a network with n users, each user is connected to (n – 1) others. This suggests a total number of connections of n • (n – 1). However, each connection is counted twice, so the number of unique connections is n • (n – 1) ÷ 2.
  In a network with n people, how many connections does each person have?
  What does this suggest about the total number of connections in the network?
  If I shake hands with you, you're also shaking hands with me. How many times is this handshake counted in the total?
  *What should we do to the total number of handshakes to get the number of **unique** handshakes?*

3 Write an equation for the number of possible connections for *u* users, and graph it below. As the network grows, how does the number of additional connections change, and how might this affect the user experience?



20 users: (20 • 19) ÷ 2 = 190 connections
100 users: (100 • 99) ÷ 2 = 4950 connections

Connections for u users:

$$C = \frac{n \times (n-1)}{2} = \frac{n^2 - n}{2} = 0.5n^2 - 0.5n$$

The larger the network, the more connections will be added with each new user. When the network is big (e.g. 100 users), a new user will become connected to lots of existing users (99), but they may not notice him much (+ 1%). On the other hand, when the network is small (e.g. 10 users), a new user will connect to fewer people (9), but they may feel his presence more (+ 10%).

Key Insights & Guiding Questions

- In a network with *n* users, each user is connected to (n – 1) others, implying a total of n • (n – 1) = n^2 – n connections. Since each of these are counted twice, though, the number of unique connections is (n^2 – n) ÷ 2 = 0.5n^2 – 0.5n.
  How did you find the number of unique connections in a network with 100 people?
  Based on this, how many unique connections must there be in a network with n people?

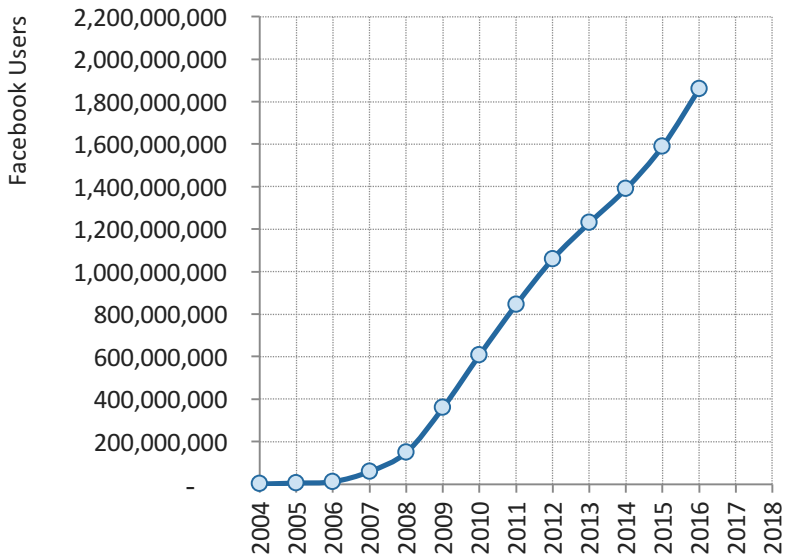
- As the number of users increases, the total number of connections increases by more and more. This can be seen in the shape of the upwards-curving graph.
  As the number of users increases, how does the number of connections change? How does this graph show this?

Deeper Understanding Questions

- *When would you say an additional user has a bigger impact: when the network is small or when it is big?*
  It depends on how you measure impact. In absolute terms, an additional user contributes more connections when the network is big. In terms of a percent, an additional user contributes more when the network is small.

- *Instead of n(n – 1) ÷ 2, could we just write the connections as n^2 ÷ 2?*
  For 100 users, the actual number of connections is 100 • 99 ÷ 2 = 4950, compared to 100^2 ÷ 2 = 5000. This isn't a huge difference. And the larger *n* gets, the more negligible the difference becomes.

4 In 2004, Mark Zuckerberg launched Facebook from his dorm room at Harvard. Today, approximately one in four humans uses it each month. The following graph shows how the number of Facebook users has grown over time.



Year	Users	Connections
2004	1 million	5.0×10^{11}
2005	5.5 million	1.5×10^{13}
2006	12 million	7.2×10^{13}
2007	60 million	1.8×10^{15}
2008	150 million	1.1×10^{16}
2009	360 million	6.5×10^{16}
2010	608 million	1.8×10^{17}
2011	845 million	3.6×10^{17}
2012	1.06 billion	5.6×10^{17}
2013	1.23 billion	7.6×10^{17}
2014	1.39 billion	9.7×10^{17}
2015	1.59 billion	1.3×10^{18}
2016	1.86 billion	1.7×10^{18}

a. How has the number of connections changed between when you first heard of Facebook and now, and what might be some upsides of this?

At the end of 2005, there were 1.5×10^{13} possible connections. At the end of 2016, there were 1.7×10^{18}. This means there were roughly 10^5, or 100 thousand, times as many connections in 2016 than in 2005. Today, users are exposed to many more people and ideas.

b. How do you expect the number of actual connections on Facebook compares to the number of possible connections? Explain.

Facebook is divided into groups of "friends." Even though there are more than 1.5 billion users, users aren't all connected to one another. This means that the number of actual connections is far smaller than the number of possible connections.

Teaching Tip

When students find the number of connections, their calculators will likely return the results in scientific notation. For 2004, (1,000,000 • 999,999) ÷ 2 = 5.0 E11. You may need to remind students what this means: 5.0 times ten *eleven times*, i.e. 5 followed by 11 zeroes = 500,000,000,000. You may also need to remind students the names of very large numbers: 10^9 = billion, 10^{12} = trillion, 10^{15} = quadrillion, 10^{18} = quintillion, 10^{21} = this-is-crazy-illion. (Okay, we made the last one up.)

Key Insights & Guiding Questions

- The number of Facebook users has grown dramatically since it was founded. It's an absolutely gargantuan network.
  What do you notice about how Facebook has grown? At this point, could another social network even compete?
- There's a difference between possible connections and actual connections. Because of how Facebook works, the number of actual connections is smaller than the number of possible connections.
  If you have a Facebook account, are you connected with every other user?
  How many people are you connected with, and how many actual connections does this imply?

5 For many users, Facebook is the main source of news and information. Some people think it exposes us to new and diverse ideas. Others think it's an "echo chamber" where we only hear perspectives we already agree with. In your experience, does Facebook cause us to become more or less open-minded, and how well do you think it's meeting its goal of "bringing the world closer together?"

Answers will vary. Sample response:

Even if people use Facebook in such a way that only reinforces what they already believe, this was already happening with cable TV, conservative and liberal blogs, etc. On the other hand, while ideological segregation has always been possible, it's become a lot easier on Facebook. Whereas we have to actively buy a newspaper or tune to a TV channel, Facebook delivers everything straight to our news feeds, and it's constantly updating its algorithm to deliver us exactly what it thinks we want to see…which in many cases means exactly what we already believe. As more and more people rely on Facebook for news and information, then everyone may become more and more convinced that their ideas are correct. This could make it very difficult for us to solve important problems that require compromise and an understanding of alternative perspectives.

Teaching Tip

Since the 2016 presidential election, the role that Facebook plays in society has become a hot and sensitive topic, and students may have strong opinions about how news coverage (including fake news) affected their preferred candidate. Before leading this discussion, please think carefully about how – and how far – you want it to go. On one hand, you may prefer to avoid a political discussion, especially if students begin to insult one another. On the other hand, if one student argues for one position while another argues for the totally opposite position, this may be *exactly* the realization you want to elicit: that neither student may be seeing the entire picture…and that Facebook may be one reason why.

Key Insights & Guiding Questions

- Facebook exposes us to information that we might not have known about otherwise.
  What kinds of things are in your Facebook feed? Have you ever seen or learned something new on Facebook?
- At the same time, Facebook doesn't show us everything. Instead, it's designed to give us content – posts from friends, videos, articles – that it thinks we'll like based on our previous behavior.
  Do you think Facebook shows you posts from all your friends or shows you articles from every possible news site? What kind of information does Facebook include in your feed, and how do you think it decides what to show you?
- Facebook has a financial interest in keeping users on the site. It's also an important source of information and may have a civic responsibility to expose users to ideas which they may disagree with.
  If you were Mark Zuckerberg, would you add liberal stories to the feeds of conservative users (and vice-versa)? What might be some consequences if you did and if you didn't?
- Being connected to lots of different people is not the same as being connected to lots of different ideas.
  When you think about your Facebook feed, would you say it includes a broad range of ideas and opinions?