LOAN RANGER

How much do you really pay when you use a credit card?

lesson guide

A young person turns eighteen and heads off to college or perhaps starts working. He is blasted with offers of easy credit. Low introductory rates, free t-shirts and water bottles, and access to quick cash entice him to sign up. Then, he goes shopping. Without a basic understanding of how credit works, he can get thousands into debt before he even knows what hit him!

Of course, credit is a complicated topic. This lesson focuses on the basics of borrowing with a credit card including interest rates, monthly compounding intervals, and different payback options. By comparing a few different scenarios, students will understand how much an item can *really* end up costing when it's purchased with a credit card.

Primary Objectives

- Compare the consequences of not paying balances on two cards with different APRs
- Observe the effect of adding monthly compounding to the model
- Compare the effect of paying the minimum versus paying more in how much interest is paid
- Compare the effect of paying the minimum versus paying more in how long it takes to pay off the balance

Content Standards (CCSS)	Mathematical Practices (CCMP)	Materials
Algebra CED.1 Functions BF.1b, LE.2	MP.4, MP.5, MP.8	• Student handout • LCD projector • Computer speakers • Graphing calculators or spreadsheet

Before Beginning…

Students should be able to calculate a percent increase of *r*% by multiplying by 1 + *r*. Familiarity with expressing percent growth as an exponential function is helpful, but not assumed. When calculating successive balances after payments are made, students will use either their calculators or spreadsheets to carry out a recursive process. If they aren't familiar with these techniques, you will need to demonstrate.

Preview & Guiding Questions

Students watch a clip from an episode of PBS' *Frontline* called *The Card Game*. In it, students watch an interview with former CEO of Providian, Shailesh Mehta, about how Providian targeted risky customers who were unlikely to pay off their balance in full. Mehta calls them "a profitable segment of the credit card industry."

Ask students if they know why a customer's not paying off his balance would be profitable for the credit card company. It's possible some students have been educated about interest, but many may have a fuzzy picture of it at best. The goal of this conversation is for students to understand that when someone makes a purchase with a credit card, he is *borrowing* the money. "Interest" is what the company charges for the privilege of borrowing their money, and it's a percentage of the amount borrowed. Typically, if a customer repays the credit card company within thirty days, no interest is charged (though other fees may apply). The longer it takes to pay back, the more interest they can charge.

- *How does a credit card company profit when someone doesn't pay off his balance?*
- *What is "interest?" How does it work?*
- *Is it possible to use a credit card without having to pay any interest or fees?*
- *What are some ways customers could get in financial trouble with credit cards?*
- *Do you think credit card companies want customers to pay off their balances?*

Act One

By comparing a purchase made with two different cards at different interest rates, students will understand the meaning of **Annual Percentage Rate** (**APR**), and how interest accrues annually if a credit card bill is not paid. Then, they will learn that credit card companies actually calculate interest monthly by increasing the balance by 1/12 of the annual rate, and investigate the result after one year.

Act Two

In the previous act, students saw what happens when a customer doesn't pay his balance. However, most credit cards require customers to pay some **minimum payment** every month. In this act, students see the effect of making a minimum payment versus a higher amount. They will see that by making double the minimum payment, it takes much less time to pay off the loan, and they end up paying far less interest.

Act One: Something Borrowed

1 Imagine two friends each buy an iPod Touch for $300, and pay with a credit card. When this happens, the credit card company pays Apple, and the friends become indebted to the credit card company. For every year they don't pay back the debt, the company charges them interest: a percentage of what they owe. The interest rate is called an **annual percentage rate** (**APR**), while the amount owed is called the **balance**.

Using the table below, calculate how much each person would owe over time if neither made any payments to the credit card company. After six years, how much would the iPod end up costing?

		Balance After				
Customer	Annual Rate	0 years	1 year	2 years	3 years	6 years
A	18%	*$300.00*	*$354.00*	*$417.72*	*$492.91*	*$809.87*
B	36%	*$300.00*	*$408.00*	*$554.88*	*$754.64*	*$1,898.26*

Explanation & Guiding Questions

To calculate the new balance, students may calculate the interest first, then add it to the current balance. For example, to find Customer A's balance in year 1, they might first take 18% of $300 = $54…and then add this back to $300 to get $354. Alternatively, $300 + 0.18($300) = $300 + $54 = $354. This will give correct answers. However, calculating in this way, in two steps, will make it harder to generalize into an equation in the next question.

Instead of the two-step process, encourage students to perform the entire calculation in one step. To find Customer A's year 1 balance, they can factor 300 + 0.18(300) to yield 300 × (1 + 0.18), or 300 × 1.18. In other words, the new balance is 118% of the previous one: the 100% that was there before – i.e. the balance – plus the new 18% interest.

Doing the calculation in one step will allow students to more easily find consecutive balances using the recursive functionality on their calculators or spreadsheet. (For example, in a TI-84, one would type **300 Enter * 1.18 Enter Enter Enter…**, and the calculator will keep finding 18% more than the previous result.)

One error you might encounter is students' calculating the original $54 interest amount, and then using it in every subsequent year. It's important that students realize that the interest depends on the balance; since the balance increases each year, so does the interest. After the first year, the customer is paying interest on the interest!

- *How can you combine (i) multiplying 300 × 0.18, then (ii) adding this to 300, into a single step?*
- *Why is 300 + 0.18(300) equivalent to 300(1 + 0.18)?*
- *What is the new balance as a percent of the original balance?*
- *Why don't you just keep adding the original $54 every year?*

Deeper Understanding

- *How much interest does each person owe after six years? (A: $509.87, B: $1598.26)*
- *Customer B's interest rate is double Customer A's. Does he owe double the interest? (Nope. B owes about 1.88 times the interest of A.)*
- *What could a credit card user do to avoid paying interest? (Pay back the loan.)*

2 In reality, credit card companies don't charge interest every *year*; they charge interest every *month*. To determine the **monthly interest rate**, divide the APR by 12.

Complete the table below. Do you think it matters how often credit card companies charge interest? Explain.

		Balance After				
Customer	Monthly Rate	0 months	1 month	2 months	3 months	12 months
A	*1.5%*	*$300.00*	*$304.50*	*$309.07*	*$313.70*	*$358.69*
B	*3.0%*	*$300.00*	*$309.00*	*$318.27*	*$327.82*	*$427.73*

Compared to interest being calculated once a year, a customer ends up paying slightly more in one year when interest is calculated monthly. On the 18% APR card, the customer pays $358.69 - $354.00 = $4.69 more, and on the 36% card, the customer pays $427.73 - $408 = $19.73 more after one year. It makes a bigger difference for the customer with the higher APR, but since his balance is increasing so much faster, this is not so surprising.

Explanation & Guiding Questions

Before students dive into calculating, you might ask whether they think charging 1/12 of the APR, but 12 times, will result in the same balance as charging the whole APR one time. They may be surprised the result is not the same. The reason: in the second month, there is a bit of interest charged *on the interest* added during the first month (and so on, in subsequent months).

- *Do you think paying the monthly rate twelve times is equivalent to paying the annual rate once?*
- *Explain why these two methods do not result in the same balance after one year.*
- *Compared to the annual rate, how much more is each customer paying due to monthly compounding?*

Deeper Understanding

- *Do you think Customer A would notice the additional $4.69 he pays due to monthly compounding?*
- *Do you think the credit card company cares about the additional $4.69 it makes from monthly compounding? (The credit card company may not care about $4.69 from an individual. However, it may have millions of customers.)*
- *In addition to the extra revenue, why might a credit card company want to charge interest monthly? (If the credit card company charged annually, a customer could pay the loan back after, say, 11 months and not owe any interest.)*
- *Customer A's advertised APR is 18%, but he's really paying more than this due to monthly compounding. What is his* ***actual*** *annual rate due? (The customer is paying 1.5% each month. Over the course of the year, this is* $1.015^{12} = 1.1956$*, which means he's effectively paying an annual rate of 19.56%.)*
- *Should credit card companies be required to advertise the effective annual interest rate? (If a customer doesn't take the entire year to pay off the balance, then the effective annual rate doesn't matter. Also, it might be confusing to advertise the effective annual rate, since the monthly rate depends on the APR.)*

3 Write an expression for the balance after ***m*** months with an APR of ***r***. If neither friend made any payments for an entire decade, how much would the $300 iPod end up costing in total?

balance	=	$300\,(1 + r/12)^{m}$
A's balance	=	$300\,(1 + 0.18/12)^{120}$
A's balance	=	*$1790.80*

balance	=	$300\,(1 + r/12)^{m}$
B's balance	=	$300\,(1 + 0.36/12)^{120}$
B's balance	=	*$10,413.30*

Explanation & Guiding Questions

Customer A is charged 1.5% interest each month. This means that the balance is multiplied by 1.015 each month: the 100% from before (1), plus the additional 1.5% interest (0.015). To find the balance after, say, six months:

$$300 \times 1.015 \times 1.015 \times 1.015 \times 1.015 \times 1.015 \times 1.015 = 300 \times 1.015^{6}$$

To find the balance after *m* months, students can generalize this to $300 \times (1.015)^{m}$. So what about the interest rate? Where does the 1.015 come from? This is just the APR (18%) divided by 12. Thus we could rewrite the balance as:

$$300\left(1 + \frac{0.18}{12}\right)^{m}\text{, or more generally as } 300\left(1 + \frac{r}{12}\right)^{m}$$

Note: Some students may interpret r = 18% as the integer 18, and write r/100 in their equations to convert to 0.18. Remind them *r* is an interest *rate*, not an integer. Thus, r = 18% = 0.18, so they don't have to divide by 100.

- *What's an easier way to write 300 × 1.015 × 1.015 × 1.015 × 1.015 × 1.015 × 1.015?*
- *In 1.015, where does the 1 come from, and where does the 0.015 come from?*
- *How can we generalize the 0.015 part for any APR, r?*

Deeper Understanding

- *If we wanted to calculate the balance after y years, how could we adjust the expression? (After one year we'd have 12 compoundings, i.e. m = 12y. We could therefore rewrite the expression as* $300 \times (1 + r/12)^{12y}$*.)*
- *What percent of Customer B's balance goes to Apple, and what percent goes to the credit card company? (2.9% goes to Apple, 97.1% goes to the credit card company.)*
- *Do you think Customer B's balance would ever really get to $10,413.30? (This is the balance that would result if he never paid, but the credit card company would likely make him pay something each month.)*

Act Two: Payback Is a Cinch

4 In reality, customers aren't supposed to pay nothing. Instead, credit card companies typically charge a **minimum monthly payment**; if a customer doesn't pay *at least* this amount, the company may charge additional fees.

Imagine the two friends sign up for a new card with an 18% APR and a $10 minimum payment, and use this card to make the $300 purchase. If the first friend pays the minimum each month, while the second friend pays $20, what will their balances be after one month?

Customer A

1. *Balance = $300*
2. *Customer A pays $10 (balance = $290)*
3. *Interest = 1.5% of $290 = $4.35*
4. *New Balance = $294.35*

Customer B

1. *Balance = $300*
2. *Customer B pays $20 (balance = $280)*
3. *Interest = 1.5% of $280 = $4.20*
4. *New Balance = $284.20*

Explanation & Guiding Questions

Students may wonder which happens first: the payment is applied, or the interest is charged. They might also wonder which is worse. To find out:

Payment applied, then interest charged	vs.	Interest charged, then payment applied
($300 - $10)(1.015) = $294.35	<	($300)(1.015) - $10 = $294.50

Fortunately, when a customer makes a payment, the payment is applied first, *then* the interest is charged on the *remainder*. This makes sense; otherwise someone who pays off 100% of his balance will still owe interest!

Even though Customer A paid $10, his balance only decreased by $5.65. Meanwhile, Customer B paid $20, and his balance decreased by $15.80. Put another way, 56.5% of Customer A's payment went towards paying down the balance, while 79% of Customer B's did. By paying a higher amount, Customer B pays off a higher percent.

- *Does it matter which is applied first: the interest charge or the payment? If so, which is worse?*
- *By how much did each customer's balance decrease when he paid $10/$20?*
- *What percent of each customer's payment went towards paying down the balance?*

Deeper Understanding

- *Do you think it's good that credit card companies charge some minimum monthly payment?*
- *Should Customer A expect it to take more than, less than, or exactly 30 months to pay off the $300 balance? (It will take more than 30 months. Even when Customer A paid $10, the balance only decreased by $5.65.)*
- *How long do you estimate it will take each customer to pay off his balance? (Answers will vary.)*
- *How much do you estimate each customer will end up spending? (Answers will vary.)*

5 Assume that both friends continue to make the same monthly payments as before. Calculate how much each person will have **paid in total** and what their **balances** will be after one, two, three, and twelve months.

After a year, how much will each customer have spent…and by how much will he have reduced his balance?

	0 months	After 1 month		After 2 months		After 3 months		After 12 months	
	Balance	Total Paid	Balance	Total Paid	Balance	Total Paid	Balance	Total Paid	Balance
A	*$300*	*$10*	*$294.35*	*$20*	*$288.62*	*$30*	*$282.79*	*$120*	*$226.32*
B	*$300*	*$20*	*$284.20*	*$40*	*$268.16*	*$60*	*$251.89*	*$240*	*$93.95*

After one year, Customer A will have reduced his balance by $300 – $226.32 = $73.68, and spent $120 to do this.
After one year, Customer B will have reduced his balance by $300 – $93.95 = $206.05, and spent $240 to do this.

Explanation & Guiding Questions

The technique used in an earlier question for applying operations recursively in a calculator or spreadsheet will come in handy here. On a TI graphing calculator:

Step	Keystroke	Result	Explanation
i.	**300 Enter**	300	Original balance
ii.	**(Ans – 10) * (1 + 0.18/12) Enter**	294.35	Balance after 1 month
iii.	**Enter**	288.62	Balance after 2 months
iv.	**Enter**	282.79	Balance after 3 months
v.	**Enter**	276.89	Balance after 4 months
vi.	**etc.**	---	Balance after *m* months

After following the payment plans for twelve months, the result is drastic. Customer A has paid $120 to reduce his balance by $73.68; put another way, he basically "gave" the credit card company $120 – $73.68 = $46.32 for the privilege of borrowing its money.

Meanwhile, customer B paid $240 to reduce his balance by $206.05. Even though he was making higher payments, he only "gave" the credit card company $33.95. This demonstrates how expensive it is to borrow money from the credit card company, especially if you make the minimum payment each month.

- *How can we use technology to automate repetitive operations?*

Deeper Understanding

- *If Customer A spent $120 to reduce his balance by $73.68, where did the rest of the money go? (To the credit card company!)*
- *At the end of the year, who do you think is better off: Customer A or Customer B? (On one hand, Customer B spent twice as much: $240 vs. $120. On the other hand, he reduced his balance by almost three times as much: $206.05 vs. $73.68.)*

6 Calculate how long it will take each friend to pay off his debt, and how much he'll end up paying in total for the $300 iPod. Based on this, how much would you say an item really "costs" when you pay with a credit card?

Person A: Pays off debt in the 40th month, with 39 payments of $10 and a final payment of $3.49. He ends up spending $393.49 for the $300 iPod, so he paid $93.49 in interest.

Person B: Pays off the debt in the 17th month, with 16 payments of $20 and a final payment of $16.67. He ends up spending $336.67 for the $300 iPod, so he paid $36.67 in interest.

Explanation & Guiding Questions

Doing this by hand will be a huge pain. Fortunately, if students follow the calculator steps from the previous question, they should have a relatively easy time determining how many months it'll take for each customer to pay off his balance; they just need to count the number of times they press "Enter" after entering the original $300 balance. Once the balance falls below the monthly payment, then each customer will just have one more payment to go. For instance, after 39 months, Customer A's balance will be $3.49, which is below his $10 monthly payment. This means he'll make a total of 40 payments, and will pay a grand total of (39 × $10) + $3.49 = $393.49.

The difference between $93.49 and $36.67 might not seem like such a big deal, but it is three times as much! Students should know that the average American has $4000 in credit card debt…not $300.

At the conclusion of this lesson, students should understand two ways to reduce the amount they spend in interest. First, a lower APR makes a huge difference, especially if you don't expect to pay off your entire balance each month. Second, making higher than the minimum payment can result in paying *much* less interest in the long run.

- *If Customer A ends up spending $393.49 for a $300 iPod, how much interest did he pay in total?*

Deeper Understanding

- *How did each customer's monthly payment affect how much he paid in interest? (Customer A made half the monthly payment as did Customer B, and ended up paying almost three times as much in interest.)*
- *What are some consequences of only making the minimum monthly payment? (It takes you longer to pay off the balance, and you end up paying more in total.)*
- *When might someone still want to make the minimum allowable monthly payment? (Maybe that's all they can afford. Alternatively, if they're savvy investors, maybe they can make more money elsewhere.)*
- *What are some strategies for using a credit card wisely? (Choose a credit card with the lowest interest rate possible; avoid borrowing more than you can pay off in a month; if you do carry a balance, make the highest monthly payment that you can afford; etc.)*
- *Other than the APR, what are some things to consider when choosing a credit card? (Annual fees; penalties; bonuses like miles and cash back; etc.)*